

Exponential Growth and Graphing Using Semi-log Paper

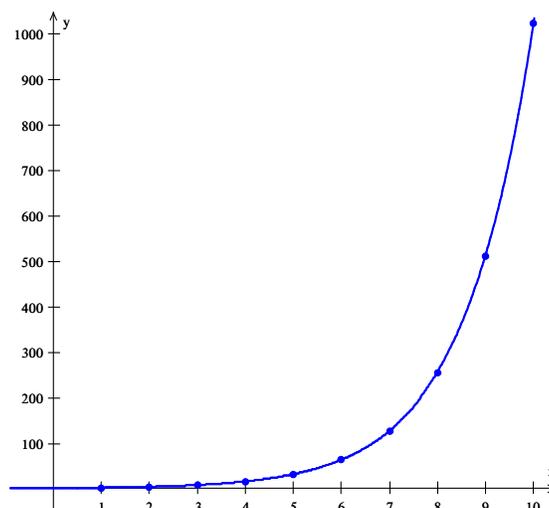
Exponential functions are used for modeling and solving many real-world problems. Examples include compound interest, population growth, and radioactive decay. For this project, you'll explore a new way to graph exponential functions using semi-log graph paper. You'll also work with an application of the exponential function for predicting growth of a bacterial culture.

Semi-Log graph paper is used for plotting systems that are modeled by exponential equations. It can be used to plot very small as well as very large numbers.

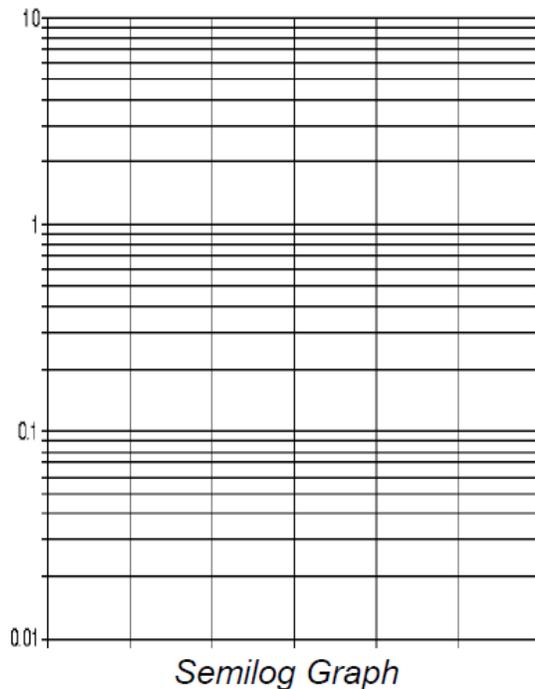
For example, let's plot the graph of $y = 2^x$ by plotting points. First we'll evaluate the function for several values of x .

x	$y = 2^x$	(x, y)
0	$y = 2^0 = 1$	(0, 1)
1	$y = 2^1 = 2$	(1, 2)
2	$y = 2^2 = 4$	(2, 4)
3	8	(3, 8)
4	16	(4, 16)
5	32	(5, 32)
6	64	(6, 64)
7	128	(7, 128)
8	256	(8, 256)
9	512	(9, 512)
10	1024	(10, 1024)

Graphing this function on a traditional Cartesian coordinate plane will require us to use a very "tall" y-axis:



Semi-log graph paper has one axis with a logarithmic scale and the other axis has a linear scale. This type of graph allows us to more easily see details for small values of y as well as large values of y . For the semi-log paper shown here, the y -axis is marked so that height represents the log of the value to be graphed. The base lines that are labeled on the y -axis represent powers of 10 or 10^n for integer numbers n . This graph is *3-cycle semi-log graph paper*. The first cycle begins with $y = 0.01$ ($y = 10^{-2}$) and ends with ten-times that or $= 0.1$ ($y = 10^{-1}$).

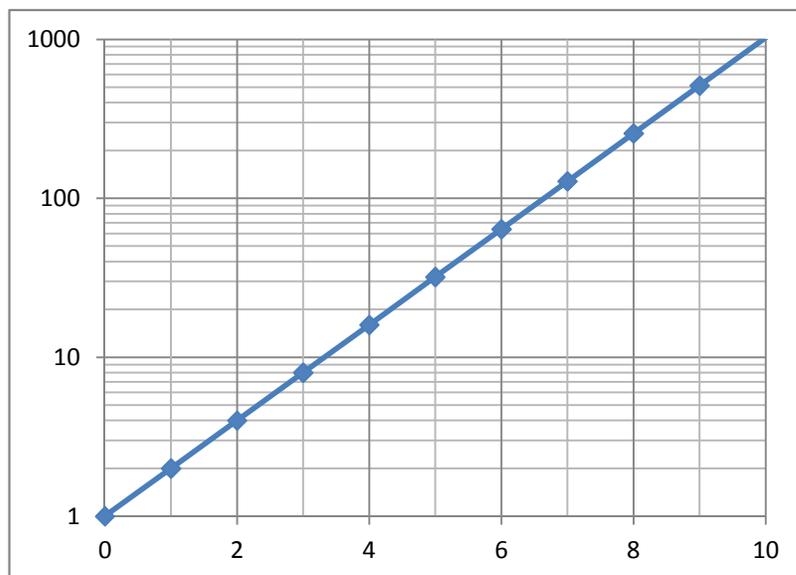


Each cycle has 10 lines, and the line immediately above the base line represents 2 times the base line. The line above that represents 3 times the base line, and so on. For the given graph, the first base line represents $y = 0.01$, so the next horizontal line above that represents $2 * y = 2 * 0.01 = 0.02$.

1. What is the domain of the logarithmic function $y = \log_a x$, for $a > 1$?
2. What is the range of the logarithmic function $y = \log_a x$, for $a > 1$?

Now let's graph $y = 2^x$ using a semi-log graph:

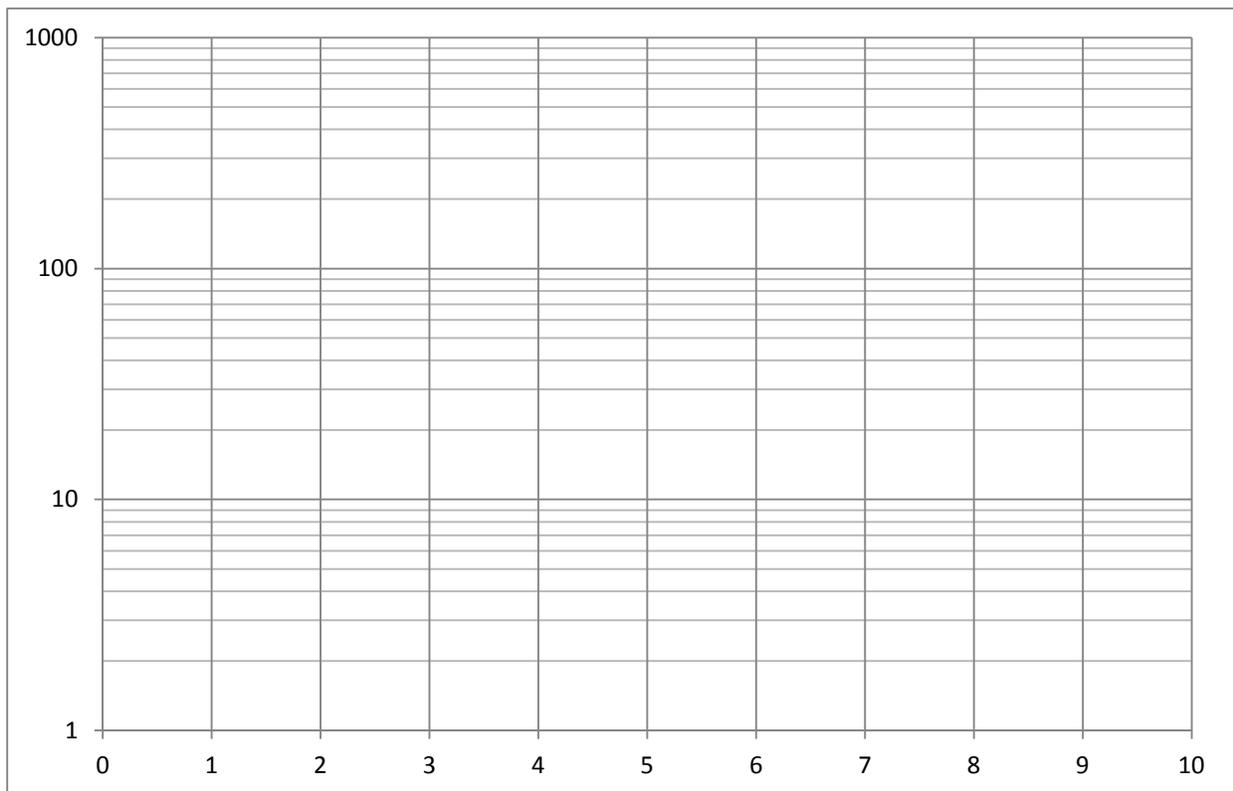
$(x, y = 2^x)$
(0, 1)
(1, 2)
(2, 4)
(3, 8)
(4, 16)
(5, 32)
(6, 64)
(7, 128)
(8, 256)
(9, 512)
(10, 1024)



3. What value of y is represented by the 4th horizontal line from the bottom? _____
4. What value of y is represented by the 12th horizontal line from the bottom? _____

Examine the given graph carefully and be sure you understand how each point is plotted.

5. Next use the semi-log graph below to plot both the graph of $y = 3^x$ and of $y = 1.5^x$. Start by evaluating the functions for each integer value of x from 0 to 10, then plot the points on the graph below, then use a straight edge to carefully draw the lines. Label each line with the equation.



- Using complete sentences briefly compare the three semi-log graphs ($y = 2^x$, $y = 3^x$, and $y = 1.5^x$).

Exponential functions, when graphed on semi-log paper, are straight lines. We can also see this transformation algebraically.

- Consider the function $y = 3^x$. Use the properties of logarithms to write $\log y$ as a linear function of t . (i.e. In the form $\log y = mx + b$.) Carefully show your work.

- Using your equation from #7, report the exact slope of your line and also the decimal approximation rounded to the nearest hundred-thousandth.

To find the slope using your graph, you use two points on the line (t_1, y_1) and (t_2, y_2) to find

the slope $m = \frac{\log y_2 - \log y_1}{t_2 - t_1}$

- Using two of the ordered pairs that you used to plot the graph of $y = 3^x$ above, compute the slope of your plotted line, rounding to the nearest hundred-thousandth. Carefully show your work.

10. What is the $\log y$ intercept of your linear equation from #7? (Report an ordered pair.)

11. What is the $\log y$ intercept on your graph of $y = 3^x$ above? (Report an ordered pair.)

12. Compare your results for #8 and 9 and for #10 and 11 – what do you find?

13. Similarly, write the function $y = Ce^{rt}$ as a linear function of t by using the properties of logarithms, but this time use the natural logarithm. Recall that C and r are constants. Carefully show your work.

14. What is the slope of your line?

15. What is the $\ln y$ intercept? (Report an ordered pair.)

The Scenario:

Lab technicians at Innovabio need to grow cultures of _____ bacteria in the lab. The technicians need to grow _____ amount of cells so that they can infect them with a particular virus in order to study its' properties. The technicians retrieve a test tube of the bacteria from an overnight culture. They put 1 mL of the bacteria broth into a Liter flask of broth to grow. The flask is placed into a shaking water bath that is kept at a constant temperature. As the bacteria multiply, the liquid in the flask progressively becomes more and more opaque. The technicians use a spectrophotometer to determine optical density by measuring the amount of light that

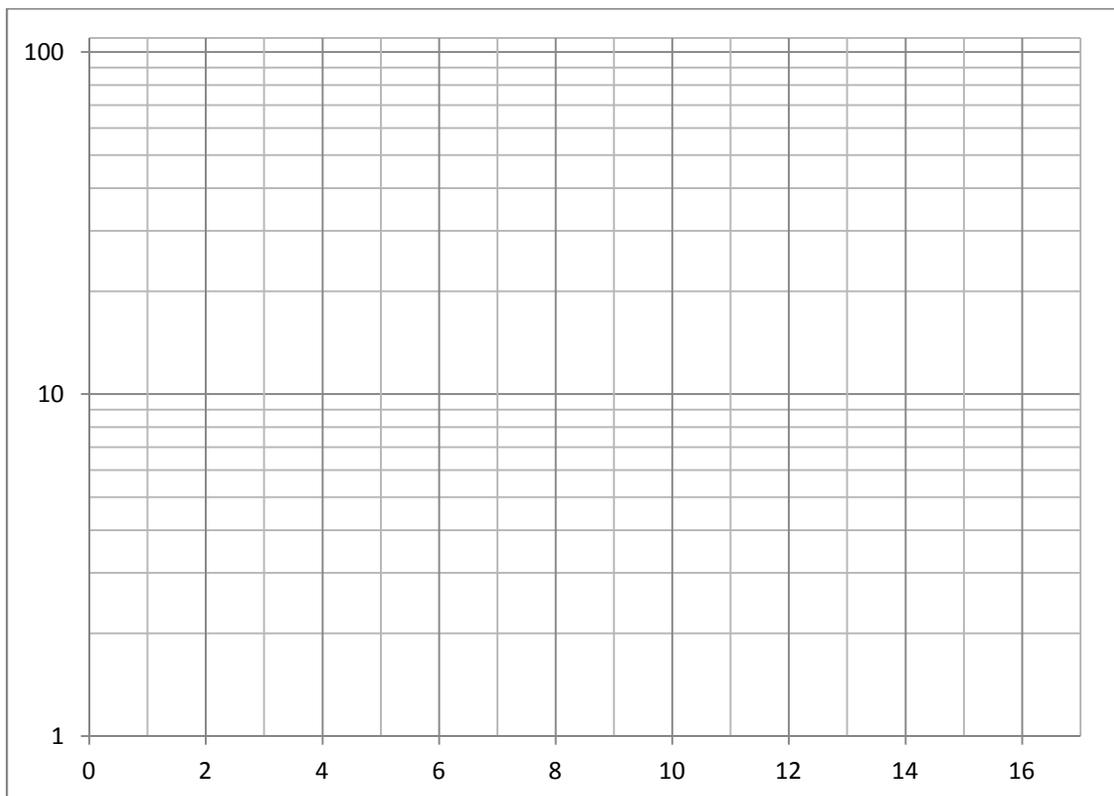
passes through the liquid in the flask. They then estimate the density of bacteria based on the optical density measurements. After recording for several hours, they have the following measurements:

Time (hours)	Number of Bacteria (1000s per mL)
2.5	10.07
2.8	13.07
5.4	14.59
6.5	20.70
9.2	27.94
9.5	31.50
11.0	40.04
13.3	49.90
14.6	64.72
16.4	75.57

(sample data is

from: http://commons.bcit.ca/math/examples/food/logs_exponentials/index.html)

16. Plot your observed data on the semi-log graph below.



17. Does the data appear to be approximately linear on the semi-log graph?

18. Use a straight-edge to draw a line that best fits your data on the plot above.

19. Using two points on your line, find the slope of your line. (Remember to use the \ln of the y value.) Show your work here and report the exact slope.

$$m = \underline{\hspace{2cm}}$$

20. Now find the equation of the line as a linear function of t . (Remember to use the \ln of the y values.) Show your work here and report the equation with exact values for the coefficients. (Hint: you'll need to use point-slope form of a line.)

$$\ln y = \underline{\hspace{2cm}}$$

21. Report the equation of the line with coefficients rounded to the nearest ten-thousandth.

$$\ln y = \underline{\hspace{2cm}}$$

22. Look back at your work on numbers 13 through 15. Notice that the slope of the line is r in the $y = Ce^{rt}$ format. What is r ? _____
23. Number 15 indicated that the $\ln y$ intercept is $\ln C$. Use the y -intercept from your line to find C . C is _____.
24. Now write the function in exponential format, as $y = Ce^{rt}$.
25. What does t represent in your equation?
26. What does y represent in your equation?
27. Using your equation, predict the number of bacteria in the culture after 12 hours. Show your work.
28. Compare your prediction to your graphed line – does it appear that your equation works well for prediction?
29. At approximately what time will you have 80,000 _____ bacteria in your culture? Show your work.
30. Does your answer seem reasonable when comparing it to the original data?

Reflection:

Each individual in a group must submit their own reflection. It must be typed with correct spelling and grammar. Your reflective writing should be approximately $\frac{1}{2}$ of a page in length.

31. Give some examples of exponential growth or decay problems in applied science fields. Did this project change the way you think about how math can be applied to the real world? Write one paragraph stating what ideas changed and why. If this project did not change the way you think, write how this project gave further evidence to support your existing opinion about applying math. Be specific.